1. Find the lengths and the inner product of $x = (1, 4, 0, 2)$ and $y = (2, -2, 1, 3)$.

$2 - 8 + 6 = 0 = x^T y$

$\sqrt{1 + 16 + 4}$

$\sqrt{21} = \|x\|$

$= \sqrt{x^T x}$

$\sqrt{4 + 4 + 1 + 9}$

$\sqrt{18}$

$= \|y\|$

5. Which pairs are orthogonal among the vectors v_1, v_2, v_3, v_4?

$$v_1 = \begin{bmatrix}1\\2\\-2\\1\end{bmatrix}, \qquad v_2 = \begin{bmatrix}4\\0\\4\\0\end{bmatrix}, \qquad v_3 = \begin{bmatrix}1\\-1\\-1\\-1\end{bmatrix}, \qquad v_4 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}.$$

$$V_1^T V_3 = [1\ \ 2\ -2\ \ 1]\begin{bmatrix}1\\-1\\-1\\-1\end{bmatrix}$$

$$= (1-2+2-1] = 0$$

$$V_2^T V_3 = [4\ 0\ 4\ 0]\begin{bmatrix}1\\-1\\-1\\-1\end{bmatrix}$$

$$= (4-4] = 0$$

R^2

$$V_1^T V_2 = 0$$

$$V_3^T V_2 = 0 \Rightarrow V_1 = aV_3$$

$V_1 = \begin{bmatrix}a_1\\a_2\end{bmatrix}$ $\qquad a_1b_1 + a_2b_2 = 0 = V_1^T V_2$

$V_2 = \begin{bmatrix}b_1\\b_2\end{bmatrix}$ $\qquad c_1b_1 + c_2b_2 = 0 = V_3^T V_2$

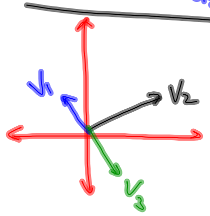
$$a_1 = -\frac{b_2}{b_1} a_2$$

$V_3 = \begin{bmatrix}c_1\\c_2\end{bmatrix}$ $\quad a_1$ vs c_1 $\qquad c_1 = -\frac{b_2}{b_1} c_2$

a_2 vs c_2

$$c_2 = k a_2$$

$$c_1 = -\frac{b_2}{b_1} k a_2$$



$$\begin{bmatrix}a_1\\a_2\end{bmatrix} = \begin{bmatrix}-\frac{b_2}{b} a_2\\ a_2\end{bmatrix}$$

$$\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}-\frac{b_2}{b_1} k a_2\\ k a_2\end{bmatrix} = k\begin{bmatrix}-\frac{b_2}{b_1} a_2\\ a_2\end{bmatrix}$$

$$= k\begin{bmatrix}a_1\\a_2\end{bmatrix}$$

If V_1, V_3 are vectors

&

$V_1 = aV_3$

then

$V_1^T = aV_3^T$

Then if $V_1^T V_2 = 0$,

$aV_3^T V_2 = 0 \Rightarrow V_3^T V_2 = 0$

7. Find a vector x orthogonal to the row space of A, and a vector y orthogonal to the column space, and a vector z orthogonal to the nullspace:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}.$$

$$\begin{bmatrix} 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$